Technical Notes

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Algorithm for Inviscid Flow Using the Viscous Transonic Equation

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Introduction

THE inviscid flow over a thin supercritical symmetric airfoil is studied using an algorithm employing Sichel's[1] "viscous transonic equation" (VTE). The formulation used formally eliminates the need for mixed differencing, shock-point, and parabolic operators, although, in a sense, the method is similar to existing ones. The usual approach for steady, small-perturbation transonic flows deals with an inviscid equation that changes type at sonic lines and shocks and that forces the coefficient of the φ_{xx} term there to vanish. At these points, special parabolic and shock-point operators are used, in addition to mixed, conservative differencing; these insure that shocks with the correct jumps appear. This supplementary information is required because high-order derivatives involving viscous terms have been omitted. (A similar situation arises in shear flow stability, when the inviscid Rayleigh equation, which breaks down at the critical layer, requires additional information as would be furnished by the more complete Orr-Sommerfeld description.)

In this Note, a high-order equation is used which accounts for the effects of compressive viscosity; the equation describes inviscid flow only, but it implicitly contains the correct jump conditions. However, because the governing equation is now formally parabolic, the same difference formulas can be applied everywhere. This approach was discussed recently by the author,[2] and good qualitative agreement with Martin's[3] results was obtained. The computation time for convergence, though, was excessively long. In this Note, a different procedure is described which represents a significant improvement over the author's previous scheme. (The method in its present form is still less efficient than existing Murman-Cole schemes, however.) One advantage of the method appears to be in the simpler required program logic, and further exploratory studies currently are being pursued along the lines outlined in this Note.

Analysis

The nondimensional VTE for the perturbation velocity potential φ is $\epsilon\varphi_{xxx} + (K-\varphi_x)\varphi_{xx} + \varphi_{yy}(x,y) = 0$, and it is supplemented by the boundary conditions $\varphi_y(x,0) = f'(x)$, $f'(x)$ being the airfoil slope, and the usual regularity conditions. Here, K is the inviscid transonic similarity parameter defined by $K = (1-M_\infty^2)/[\tau^{2/3}(\gamma+1)^{2/3}M_\infty^{4/3}]$, ϵ is a small positive number proportional to an inverse Reynolds number based on the compressive viscosity, M_∞ is the subsonic freestream Mach number, τ is half the thickness ratio, and γ is the ratio of specific heats; the pressure coefficient is then given by $C_p = -2\tau^{2/3}\varphi_x/[(\gamma+1)^{1/3}M_\infty^{2/3}]$. The vanishing of $K-\varphi_x$ at sonic lines and shocks suggests horizontal line relaxation, so that we "sweep" the computation box (Fig. 1) upwards repeatedly until convergence. The analysis proceeds by introducing the three-component vector $(\varphi, U, W) = (\varphi, \varphi_x, \varphi_{xx})$ at each grid point. Then, along each horizontal line where j is fixed, nonlinearly coupled algebraic equations in φ, U, and W must be solved which are obtained by varying from 2 to i_{max}, the index i in the finite-difference equations to be derived. The key idea consists in approximating U, W, and φ in $\epsilon W_x + (K-U)W + \varphi_{yy} = 0$ at $(i-½, j)$ by their averaged values at (i,j) and $(i-1,j)$, and W_x by $(W_{i,j} - W_{i-1,j})/h_i$. Next the φ_{yy} term is approximated by central differences about the line $j = \text{const}$, and in the resulting formula the explicit presence of U and φ is eliminated using the trapezoidal rule, that is,

$$\varphi_{i,j} = \varphi_{1,j} + \frac{1}{2}\sum_{n=2}^{n=i} h_n(U_{n,j} + U_{n-1,j})$$

and

$$U_{i,j} = U_{1,j} + \frac{1}{2}\sum_{n=2}^{n=i} h_n(W_{n,j} + W_{n-1,j})$$

For example, one obtains for the horizontal lines in $2 < j < j_{max} - 1$

$$\begin{aligned}
&\frac{\epsilon}{h_i}(W_{i,j} - W_{i-1,j}) + \frac{h_i^2}{4(g_{j+1}+g_j)}\left[\frac{W_{i,j+1} + W_{i-1,j+1}}{g_{j+1}}\right.\\
&\left.+\frac{W_{i,j-1} + W_{i-1,j-1}}{g_j}\right] + (W_{i,j} + W_{i-1,j})\left[\left(\frac{K-U_{1,j}}{2} - \frac{h_i^2}{4g_{j+1}g_j}\right)\right.\\
&\left.-\frac{1}{8}h_i(W_{i,j} + W_{i-1,j}) - \frac{1}{4}\sum_{n=2}^{n=i-1} h_n(W_{n,j} + W_{n-1,j})\right]\\
&+\sum_{n=2}^{n=i-1} h_n\left[\frac{1}{2}(h_n + h_i) + \sum_{K=n+1}^{K=i-1} h_K\right]\left[\frac{W_{n,j+1} + W_{n-1,j+1}}{g_{j+1}(g_{j+1}+g_j)}\right.\\
&\left.-\frac{W_{n,j} + W_{n-1,j}}{g_{j+1}g_j} + \frac{W_{n,j-1} + W_{n-1,j-1}}{g_j(g_{j+1}+g_j)}\right]\\
&+\frac{2\varphi_{1,j+1} + U_{1,j+1}\left[h_i + 2\sum_{n=2}^{n=i-1} h_n\right]}{g_{j+1}(g_{j+1}+g_j)}\\
&+\frac{2\varphi_{1,j-1} + U_{1,j-1}\left[h_i + 2\sum_{n=2}^{n=i-1} h_n\right]}{g_j(g_j+g_{j+1})}\\
&-\frac{2\varphi_{1,j} + U_{1,j}\left[h_i + 2\sum_{n=2}^{n=i-1} h_n\right]}{g_{j+1}g_j} \equiv 0
\end{aligned}$$

For the line $j = 2$ adjacent to the airfoil, the foregoing equation can be modified for the required tangency condition, and a similar equation is easily derived. (Similar considerations hold for $j = j_{max} - 1$.)

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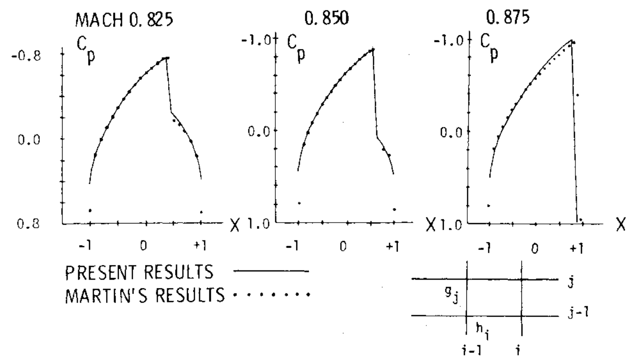

Fig. 1 Surface pressure comparison for 10% thick parabolic arc at sweep 200.

It is useful to examine how the present algorithm fits into the scheme of existing methods. For example, Taylor expansion of the hyperbolic difference form of the usual inviscid equation leads to a differential equation that locally behaves like the VTE, the truncation error term being similar to the viscous diffusion term. (A fully conservative, mixed differencing scheme, including parabolic and shock-point forms of the equation, is given by Murman.[4]) All of these forms can be expanded in Taylor series and can be shown to produce a term analogous to the viscous diffusion term in the truncation error, except that the elliptic and parabolic operators can be regarded as having a zero viscosity coefficient to within second-order accuracy. The key difference between these and the present method is the manner in which diffusion is added to the inviscid equation: a positive ϵ is used everywhere, which formally eliminates the need for mixed differencing. (Thus, one difference formula only is used along a row.) In the numerical work, ϵ is chosen at convenience, for stability reasons. (This does not affect the magnitude of the required jumps, although it does affect the shock thickness obtained.) In this sense, the viscosity is an artificial one, although, as noted, the VTE itself is obtainable through a formal limiting procedure applied to the full viscous equations. The foregoing nonlinear algebraic equations, arranged sequentially with i, are now solved by Newton's iteration with quadratic convergence. The corresponding derivative matrix can be shown to contain one upper codiagonal resting on a lower triangular matrix, with all matrix elements being nonzero. Thus, rapid triangularization is possible, choosing as pivots the successive elements of the upper diagonal, followed by the usual solution by direct elimination. The latest updated values of W are always used in the relaxation procedure, the computations are initialized with $W=0$, and, in the results that follow, we assume that $U_{I,j}=\varphi_{I,j}=W_{i\max,j}=\varphi_{i,j\max}=0$.

Sample Calculations

Calculations carried out for a symmetrical, nonlifting, supercritical 10% thick parabolic arc airfoil are compared with some results of Martin.[3] Twenty uniform grids were assumed over the airfoil and ten for each direction off the airfoil. These off-airfoil horizontal spacings were stretched by a factor of 1.2 each successive grid. Twenty vertical grids were taken. The first two were 2% of chord, and the remainder were stretched by the same 1.2 factor. This resulted in an approximate 4×3 chord computation box. For $\gamma=1.4$ and $\tau=0.05$, three values of M_∞ were considered: 0.825, 0.850, and 0.875, which correspond, respectively, to $K=1.696$, 1.417, and 1.151. The assumed viscosity was 0.0175 throughout, and values of the surface pressure coefficient were obtained by second-order differences.

The calculations were performed on the CDC 7600, and convergence was achieved after 200-250 sweeps of the flowfield. (In some cases, the computations were carried out to 1000 iterations, with only 2-3% fluctuations in surface C_p values.) Figure 1 shows the good qualitative agreement achieved. Numerical experiments showed that increased values of viscosity tended to "smear" the resulting shocks, as expected. The time required per sweep in the relaxation procedure was 0.066 seconds (this is seven times faster than the original algorithm[2]) and is approximately independent of sweep number. The present method also requires only half as many sweeps for convergence, possibly because W is a smoother dependent variable. The shape of the C_p curve and the shock position appeared to be well established by the 150th sweep.

Summary and Conclusions

The scheme presented in this Note introduces artificial viscosity everywhere and formally eliminates the need for mixed differencing. (This idea was first given independently in Ref. 5.) The small disturbance equation used has the form of the VTE, although artifically large values of the viscosity are needed for numerical stability; thus, the computed results, with $\epsilon=O(h)$, are only first-order accurate for the inviscid solution. Also, because of the large resulting matrices, the method is not as efficient as current methods.

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Notes on the Flow Near a Wall and Dividing Streamline Intersection

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THE flow of an incompressible viscous fluid with negligible inertia forces near a corner between two plane boundaries was discussed by Moffatt.[1] It is found that, when either or both of the boundaries is rigid and the corner angle is less than a critical value, the flow consists of a sequence of eddies of decreasing size and rapidly decreasing intensity as the corner is approached. Detailed investigations of two-dimensional corner flow also were made by Lugt and Schwiderski.[2]

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